

## (1) Large deformations diffeomorphic metric mapping (LDDMM)

- Diffeomorphism that minimizes energy  $E(v)$  between images  $I_0$  (source) and  $I_1$  (target):

$$E(v_0) = \langle Lv_0, v_0 \rangle_{L^2} + \frac{1}{\sigma^2} \|I_0 \circ (\phi_1^v)^{-1} - I_1\|_{L^2}^2,$$

- s.t. Euler-Poincare differential equation (EPDiff):

$$\partial_t v_t + ad_{v_t}^\dagger v_t = 0 \text{ in } \Omega \times (0, 1],$$

- High computational complexity. Current deep-learning approaches are either supervised or non-geodesic.

## (2) Adjoint Jacobi Fields

- Computing  $\nabla_{v_0} E(v_0)$  is costly, but computing  $\nabla_{v_1} E(v_0)$  is straightforward.

- Perform Parallel Transport with Reduced Adjoint Jacobi Fields:

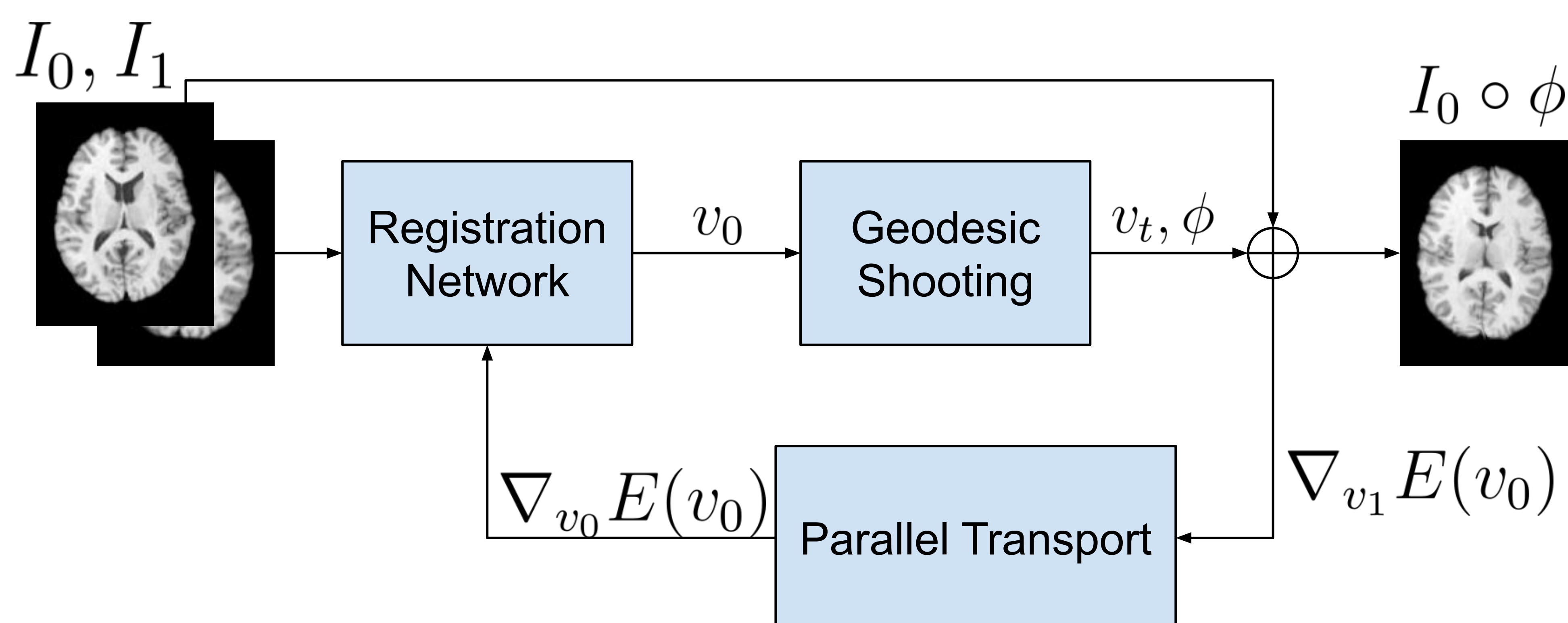
$$\partial_t U_t + ad_{v_t}^\dagger U_t = 0 \text{ in } \Omega \times (0, 1],$$

$$\partial_t w_t - ad_{v_t} w_t + ad_{v_t}^\dagger v_t + U_t = 0 \text{ in } \Omega \times (0, 1],$$

- Final gradient expression:

$$\nabla_{v_0} E(v_0) = 2v_0 + w(0).$$

## (3) Parallel Transport Registration Framework



- Registration Network predicts initial velocity field  $v_0$ .
- Use geodesic shooting to compute  $v_t$  and  $\phi$ , use parallel transport to compute final gradient  $\nabla_{v_0} E(v_0)$ .
- Back-propagate gradient through the network as usual to train it.

- Multi-resolution CNN architecture, 3 identical levels, each level takes results from the previous level:

$$L(I_1, I_0, v_0) = \sum_{\ell \in [1..3]} L_{JF}(I_1^\ell, I_0^\ell, v_0^\ell),$$

$$L_{JF}(I_1, I_0, v_0) = \langle Lv_0, v_0 \rangle_{L^2} + \frac{1}{\sigma^2} lNCC(I_1, I_w),$$

## (4) Experiments and Results

- OASIS dataset with 414 T1-weighted brain MRI images, with automatic segmentations. NIREP16 dataset 16 images, with manual segmentations.

- We obtain State-of-the-art results on two independent brain MRI datasets.

- Our method features low inference times making it a promising tool for large-scale computational anatomy studies.

