Dynamic Consensus for Deformable Body Centroid Estimation

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Abstract

We study a consensus algorithm to enable multiple independent agents to collectively estimate the centroid of a moving, deformable body. Each agent has only partial information about the system in the form of an individual reference signal. This is performed in a fully distributed manner, without centralized or hierarchical communication. We also provide a mathematical expression for the upper bound of the expected error.

Introduction

Coordination and agreement among agents are becoming a topic of growing importance given the current trend of developing and deploying an increasingly large number of autonomous agents. As scale and autonomy of these systems increase, the challenge of enabling effective consensus becomes even more important. In many real-world situations, it is unrealistic to expect every agent to directly perceive the entire system or communicate with all other agents. Each agent works with limited information, based on its surroundings and the few agents it can actually communicate with.

The coordination and consensus problem spans numerous domains, being applicable to robotics, autonomous vehicle fleets, power distribution and grid management, as well as edge computing for the Internet of Things and social interactions [1-4]. Finding efficient and stable ways to reach consensus in these kinds of decentralized systems is vital in order to ensure reliable, autonomous operations to scale in these complex environments [5].

A motivational example of a possible application is shown in (*Figure 1, left*). Estimating the origin and evolution of an oil spill with a set of drones. Using just one drone is unfeasible due to the size of the time-varying oil spill. However, by leveraging several drones and allowing them to communicate their measurements and reach an agreement, a consensus could be reached regarding the state of the oil spill.

The dynamic consensus problem

We explore dynamic consensus to reach an agreement regarding the centroid of a deformable moving body $\boldsymbol{x_c}$. We model the deformable body as a centroid surrounded by a set of points that freely oscillate around their own position and follow the translation of the centroid. Each agent follows a signal $\boldsymbol{u}_i$ which corresponds to its incomplete knowledge of the system. In our system, each of the N agents follows a signal $\boldsymbol{u}_i$, which corresponds to the position of whichever delimiting points the agent is able to observe.

The goal is then to leverage the communication between agents -codified through the Laplacian matrix $\boldsymbol{L}$ of the communication graph- so they all reach an agreement and estimate the same position of the centroid. In order to do so, a robust-to-initialization algorithm is employed:

$$
\begin{aligned}
\dot{\boldsymbol{q}} &= -\boldsymbol{Lq}, \\
\dot{\boldsymbol{x}} &= \alpha(\boldsymbol{x} - \boldsymbol{u}) - \boldsymbol{Lx} + \boldsymbol{Lq} + \dot{\boldsymbol{u}}, \\
\boldsymbol{q}&(t=0), \boldsymbol{x}(t=0) \in \mathbb{R}^{\boldsymbol{N}\times\boldsymbol{2}}, \\
\boldsymbol{q} &= [\boldsymbol{q_1}, \dots, \boldsymbol{q_i}, \dots, \boldsymbol{q_N}]^{\boldsymbol{T}}, \\
\boldsymbol{x} &= [\boldsymbol{x_1}, \dots, \boldsymbol{x_i}, \dots, \boldsymbol{x_N}]^{\boldsymbol{T}}, \\
\boldsymbol{u} &= [\boldsymbol{u_1}, \dots, \boldsymbol{u_i}, \dots, \boldsymbol{u_N}]^{\boldsymbol{T}}.
\end{aligned}
$$

This is a simplified version of the one provided in [1] since it has been assumed that the communication graph is bidirectional and the same for both sets of variables $(\boldsymbol{x}, \boldsymbol{q})$. Where $\boldsymbol{x}_i$ is the estimation made by agent i, and $\boldsymbol{q_i}$ is an auxiliary variable introduced to achieve robustness regarding initial conditions. The agents are therefore initialized randomly and are aware of incomplete information. Thanks to consensus, their estimation $\boldsymbol{x}_i$ of the centroid converges to an agreed value, that agreed value matches the true centroid $\boldsymbol{x}_c$ *(Figure 1, center)*. Given the disagreement $\|\boldsymbol{x_c} - \boldsymbol{x_i}\|$, we also compute a bound for its maximum admissible value $(e_i^{deformable})$ in the case of a deformable body building upon [1]:

$$|e_i^{deformable}(t)| \leq |e_i(t)| + \frac{\kappa||\boldsymbol{B}||}{\lambda} 2A(N-1).$$

This expression relates the error of a simple dynamic consensus $e_i(t)$, that is, signals that do not model the proposed deformable body and the second addend, that refers to the deformation error. $\kappa, \lambda, ||\boldsymbol{B}||$ are all parameters that depend on the usual dynamic consensus system. Parameter A is the amplitude of the oscillations of delimiting points and N is the number of tracking agents in our system. (*Figure 1, right)*, shows its impact on the final error as the yellow area.

We also study the final disagreement between the consensus reached by the agents and the true centroid of the body when modifying different aspects of the system. Some affect the estimation error, such as the amplitude of the oscillations or the communication graph, while other parameters like the oscillation frequencies of the delimiting points or the oscillation shape do not affect the estimation of centroid performance.

Conclusions

Several tests have been run to verify the applicability of dynamic consensus in estimating the centroid of a moving deformable body. The experimental results indicate good final estimations and a low disagreement between the true centroid and the estimated agreed centroid. However, the computed bound for the maximum possible error is quite conservative which would be non-ideal for certain applications.

The performance of the system has also been tested by changing different parameters of the system, revealing that the disagreement between the agents' consensus and the true centroid is influenced by system parameters. Some increase the error while others have no significant impact.

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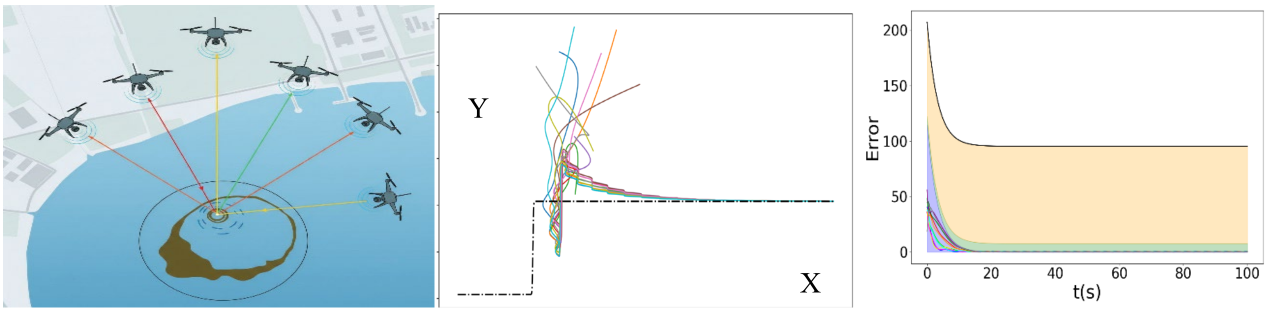

Figure 1: (left) Sketch of a possible application: A set of drones estimates the centroid of a moving and varying oil spill. **(center)** Centroid estimation estimated by each agent (solid lines) is compared against the true centroid (dashed line), demonstrating both agreement and convergence towards the true position. **(right)** Error analysis: This plot compares the error of each agent's centroid estimation (solid colored lines) with the maximum error bound (solid black line over the colored areas). From bottom to top, the two first correspond to the non deformable dynamic consensus and the upper area to the introduced term due to the deformable body.