

Model Order Reduction for Strongly Coupled Multiphysics Problems: A POD-ANN Framework and a Physics-Driven Surrogate Model

Elena Hernández-Alhambra¹, Begoña Calvo^{1,2}, Jorge Grasa^{1,2}

¹Applied Mechanics and Bioengineering.

Instituto de Investigación en Ingeniería de Aragón (I3A)

Universidad de Zaragoza, Mariano Esquillor s/n, 50018, Zaragoza, Spain.

Tel. +34-976762707, e-mail: ehalhambra@unizar.es

²Bioengineering, Biomaterials and Nanomedicine Networking Biomedical Research Centre (CIBER-BBN), Spain

Abstract

This work presents and evaluates two complementary model order-reduction strategies: POD-ANN and a physics-driven surrogate model, for strongly coupled thermo-mass-mechanical systems. Using high-fidelity cooking simulations as a case study, both approaches achieve substantial computational speed-ups while preserving dominant thermal, transport, and deformation mechanisms, enabling fast prediction and parametric exploration.

Introduction

Model order reduction (MOR) provides an efficient pathway to accelerate nonlinear multiphysics simulations that are otherwise computationally intensive. In this work, two fundamentally different MOR methodologies are developed and evaluated: (1) a projection-based POD-ANN reduced order model, and (2) a physics-driven surrogate model calibrated through Bayesian inference. Although both aim to reduce computational cost, they rely on distinct principles. One on data-driven projection of high-fidelity fields, the other on simplified mechanistic relationships and therefore offer complementary advantages. These two reduction strategies enable fast prediction, parametric exploration, and generalization both within and beyond the calibration domain.

To assess their capabilities, the two methodologies are applied to a representative food-cooking process involving coupled heat and moisture transfer, evaporation and shrinkage during pan cooking. This scenario constitutes a demanding benchmark due to its strong multiphysics coupling, nonlinear behaviour, and evolving geometry.

Material and Methods

The POD-ANN model is constructed from a database of full-order simulations, enabling fast reconstructions within the parameter space spanned by the snapshots. In this strategy [1], solutions $\mathbf{u}(\mu i) \in \mathbb{R}^n$ from varying inputs $\mu \in \mathbb{R}^p$ are collected into a snapshot matrix $\mathbf{U} \in \mathbb{R}^{n \times N}$. A singular value decomposition (SVD) of $\mathbf{U}$ yields a basis $\Phi \in \mathbb{R}^{n \times r}$ that captures approximately 95% of the variance. Solutions are projected and reconstructed as:

$$\theta^{(i)} = \Phi^T_r \mathbf{u}(\mu i) \quad \mathbf{u}'(\mu) = \Phi_r \theta'(\mu) \quad (1)$$

Using this decomposition, the coefficients $\theta^{(i)} \in \mathbb{R}^r$ are used to train a neural network that approximates the mapping $\mu \mapsto \theta$.

In contrast, the physics-driven surrogate relies on simplified constitutive laws fitted to experimental and simulated data, providing robust extrapolation and extremely low evaluation times. This surrogate model, built upon domain expertise, reduces the thermal and mechanical description to the dominant mechanisms while maintaining predictive accuracy. Thermal and moisture induced deformations are represented as:

$$\epsilon_T = \alpha (T - T_{ref}) \mathbf{I}_3 \quad \epsilon_{cw} = \beta M_w (c_w - c_{w,ref}) \mathbf{I}_3 \quad (2)$$

where ϵ_T and ϵ_{cw} are the deformations due to temperature gradient and water concentration gradient, T_{ref} and $c_{w,ref}$ are the reference temperature and water concentration which do not produce any deformation, and α and β are coefficients that govern each deformation mode. These coefficients are estimated using a Bayesian fitting framework. The total deformation is considered as the sum of these two contributions.

Model fit is evaluated using *NRMSE* (Normalized Root Mean Square Error), R^2 and a modified Akaike Information Criterion (AIC(t_c)) [2] penalizing high

computational time:

$$AIC(t_c) = n \ln(MSE) + 2 t_c \quad (3)$$

where n is the number of samples compared and t_c is the time each simulation has lasted.

Results

Reduced models' evaluation reveals their capability to accurately approximate the full-order solutions [3,4] while reducing computational cost.

For a dataset of 400 different cooking conditions, the POD model, when compared with high-fidelity model results, achieves $NRMSE$ and R^2 values shown in Table 1.

The surrogate model has been compared using a modified Akaike Information Criterion, $AIC(t_c)$, which incorporates simulation time as a relevant penalty term. $AIC(t_c)$ values for both the high-fidelity model and the surrogate are computed with respect to the experimental data, demonstrating a real speed-up in computational cost. For a specific case study, the resulting $AIC(t_c)$ comparison is shown in Figure 1.

Conclusions

Model reduction substantially reduces computational time (from 6360 to just above 60 seconds), enabling the generation of large datasets for neural network training and supporting real-time predictions. This allows broader exploration of cooking scenarios and facilitates deployment in low-resource environments. By retaining key dynamics, ROMs combined with data-driven methods offer a practical and accurate

alternative to full models, suitable for industrial food process optimization.

Acknowledgments

This work was supported by projects CPP2024-011615 LAMARR financed by the Spanish Ministry of Science and Innovation MCIN/AEI/ 10.13039/501100011033 and by the European Union “NextGenerationEU /PRTR”; and by the BSH Home Appliances Group. It has also been supported by the Department of Industry and Innovation (Government of Aragon) through the research group Grant T24-23R and the IA-COOK project.

REFERENCES

- [1]. HERTHAVEN J.S. and UBBIALI S., Non-intrusive reduced order modeling of nonlinear problems using neural networks, *J. Comput. Phys.* 363 (2018) 55–78.
- [2]. AKAIKE, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19(6), 716–723.
- [3]. HERNÁNDEZ-ALHAMBRA, E, GUÍU, P, FERRER-MAIRAL, A., MARTÍNEZ, M.A., CALVO, B., GRASA, J., and SALVADOR, M.L., 2025. Predicting optimal flipping conditions during burger pan cooking with a numerical model. *Journal of Food Engineering* 387, 112315.
- [4]. HERNÁNDEZ-ALHAMBRA, E, GUÍU, P, FERRER-MAIRAL, A., MARTÍNEZ, M.A., CALVO, B., GRASA, J., and SALVADOR, M.L., 2026. FEM modeling of hamburger pan cooking: Fat content influence and neural network-based prediction of fat loss. *Current Research in Food Science* 12, 101327.

Table 1. Error metrics for 400 different scenarios of the behaviour of the POD-ANN Framework

	Weight loss [%]	Core T [°C]	Shrinkage [%]	Pan T [°C]
NRMSE	0.0472 ± 0.0251	0.0507 ± 0.0240	0.0528 ± 0.0245	0.0307 ± 0.0200
R^2	0.9740 ± 0.0233	0.9758 ± 0.0187	0.9716 ± 0.0258	0.9116 ± 0.1031

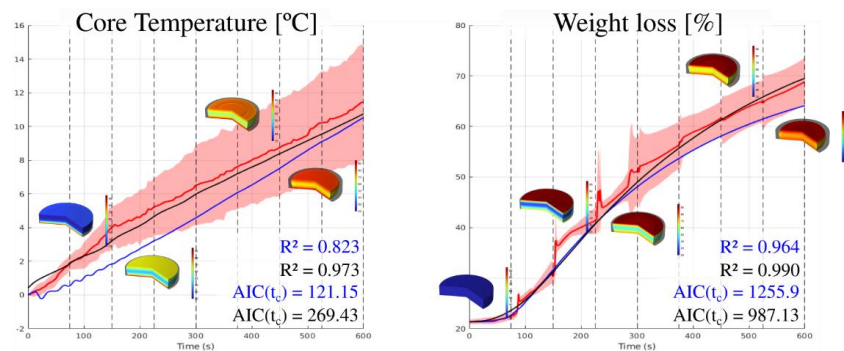


Figure 1. Experimental (red line is the mean result, shaded area corresponds to the standard deviation [3]) and both computational results, high-fidelity model (black line) and physics-based surrogated model (blue line), for a specific cooking scenario (7 flips in 600 seconds of cooking, dashed lines correspond to flips performed). Modified Akaike Information Criterion ($AIC(t_c)$) and correlation coefficient (R^2) are shown for every variable measured. Each flip shows the current temperature and the updated shape, with the initial shape rendered in a translucent black domain.