A new generation of real-time simulation techniques

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Abstract

This work presents a novel methodology for the development of real-time simulation techniques. They are focused on the PGD method, which allows the solution of complex problems by pre-calculating *all* the possible results. Subsequently, applications efficiently process these data to show particular results on demand.

PGD for real-time simulation

Introduction

Many problems in science and engineering remain intractable because of their enormous numerical complexity. In spite of the impressive advances achieved during the last decades in the fields of mechanical modeling, numerical analysis, discretization techniques and computer science, these problems are still very difficult to solve even for computers, especially if restrictive requirements are imposed, like real-time response.

In this article, the characteristics of this kind of problems will be analyzed and a new, efficient method to solve them will be studied. The key idea of this technique is that a meta-solution is pre-computed for every value of every variable in the problem prior to its implementation. Applications can subsequently access to the stored values and present the particular solutions very efficiently.

A novel approach: PGD methods

Complex problems are usually defined by functions with such a large number of parameters that they become unapproachable to solve numerically in a direct, traditional way. This phenomenon, whereby the number of degrees of freedom of the problem increases exponentially with the number of parameters, is known as the *curse of dimensionality*.

An efficient technique has recently emerged to circumvent this curse of dimensionality. It consists in using model reduction techniques to express a high dimensional function F as a finite sum of products, that is:

$$F(x_1, \dots, x_N) \approx \sum_{i=1}^{Q} F_i^1(x_1) \cdot \dots \cdot \ {}_i^N(x_N)$$

where x_i denotes a scalar or vector coordinate defined in a domain of moderate dimension—generally three. This approximation allows the reduction of the number of degrees of freedom of the problem model. For example, in mesh-based models with M nodes per coordinate, $Q \cdot N \cdot M$ degrees of freedom would be obtained, instead of the original M^N. *Proper Generalized Decomposition* (PGD) is the common name recently coined for these techniques [Chinesta, 2011].

The origin of this technique can be traced back to the LATIN method [Ladevèze, 1999], and it is seen at present as a powerful *Model Order Reduction* (MOR) method that generalizes *Proper Orthogonal Decomposition* (POD) [Chatterjee, 2000], hence its name. The solution provided by PGD methods can be considered as a meta-model, that is, a general solution for *every* value of the variables (within a given interval), which is efficiently structured in matrix form. Unlike POD, no previous computer experiment is necessary to obtain the solution.

A further step: multidimensional models

Since PGD methods are excellent to solve problems with a high number of parameters, existing models can be reformulated to incorporate unknown variables as new parameters. Thus, parametric equations can be treated as multidimensional models to take advantage of the PGD structure. Thanks to this innovative approach, all the unknown variables in real-time simulation environments (e.g. all the possible boundary conditions) can be now considered as new dimensions of the problem.

For this kind of applications, PGD methods are usually implemented in a two-stage strategy. The first one is *off-line* and the meta-solution is obtained. Although this stage may require an intensive computing power, the cost is not critical, since it is computed just once. The second stage is *on-line*, and it consists in moving from the general meta-solution to the specific values demanded for each variable. Obtaining the particular solution at this stage is almost immediate, since no high processing capabilities are required. Thus, this technique can be implemented in low-resource devices such as tablets or smartphones, expanding the range of its possible applications.

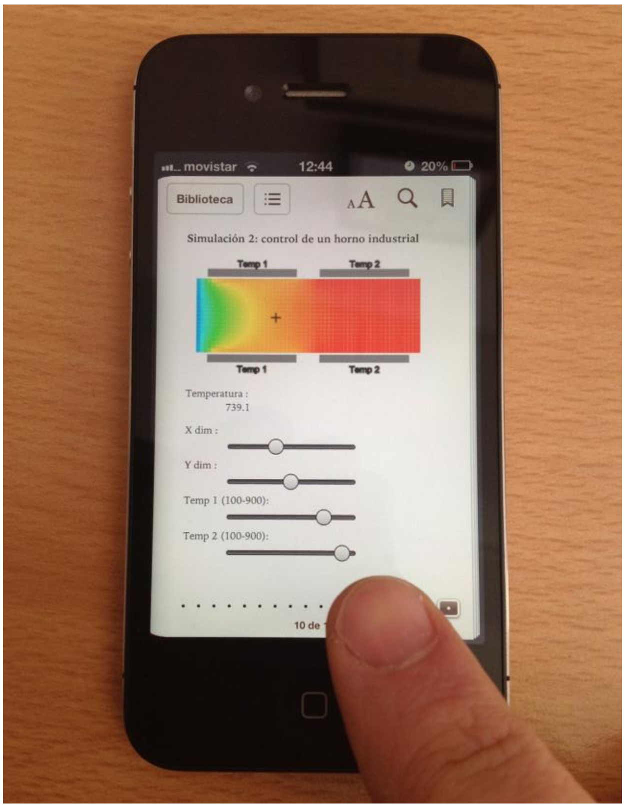

Figure 1. Real-time thermal simulation of an industrial furnace on a smartphone. The user can manually adjust the working temperature to observe the effect on the field.

Applications

Based on the two-stage strategy presented in the previous section, many challenging augmented learning tools have been developed: thermal simulation of industrial furnaces (see Figure 1), simulation of plate and shell structures for engineers, simulation of the wave propagation on a harbor, etc. The real-time capability of these applications allows their implementation in deployed, touch-screen handheld devices, or even their embedding as electronic textbooks (ePUB).

Special attention has to be paid to the development of surgery simulators with visual and haptic feedback, since it remains as an active area of research due to the complexity of the problem (see Figure 2). There are three main sources of difficulty: (1) the non-linear behavior of soft living tissues, which are frequently modeled as hyperelastic; (2) the highly restrictive feedback rates imposed by the hardware: 25 Hz for the visual display and 500 Hz for the haptic display; and (3), the nature of common surgical procedures to be implemented, like palpation, cutting or suturing.

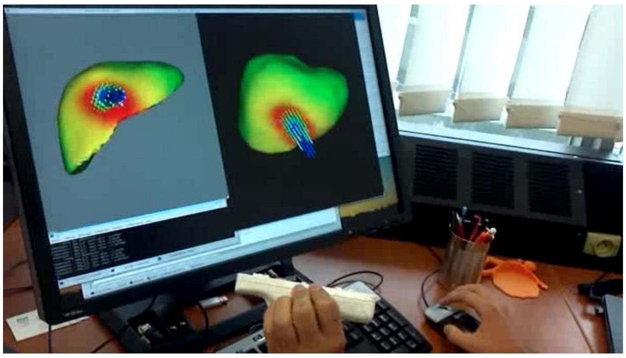

Figure 2. A PGD approach in anatomy. In this case, a human liver is being studied under palpation forces.

Conclusions

We have introduced and studied the possibilities offered by Proper Generalized Decomposition methods. The PGD method not only constitutes a new paradigm in simulation techniques, but also possesses outstanding features in the field of augmented learning. Notably, we have explored its ability to provide efficient solutions (in terms of computing power) to complex problems that have remained unapproachable for traditional simulation techniques at real-time feedback rates.

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