Structural Identifiability Applied to a Heat Transfer System

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Abstract

Identifiability is an essential property of a dynamical model whose study should be addressed before performing any parameter estimation procedure. In this work, we study the structural identifiability of a heat transfer system by making use of the local state isomorphism theorem for two scenarios based on the available experimental measurements.

Introduction

Reducing energy consumption is one of the main problems to be solved by human beings. Most of the energy consumed is linked to thermal processes. One of the keys to reducing energy consumption is to improve the efficiency of these systems. In this sense, building a dynamical model is a useful tool to improve the efficiency. The building of a model aimed at energy improvement is a critical aspect that depends on both the degree of detail and the information used in the construction process. The optimal solution is probably a hybrid modeling between models based on basic principles whose parameters are characterized from observations of the system. This modeling approach is called grey-box (see [1]-[2]) and requires identification procedures in order to estimate the set of partially or totally unknown parameters. Both if the parameters to be estimated have physical significance, or if the model is to be used to predict the dynamics of state variables that cannot be directly measured, it is essential to perform a preliminary identifiability analysis of the proposed parametric structure. Despite the importance of identifiability, the analysis of this property has been largely overlooked in the vast majority of works on dynamic modeling.

Dynamical Model

The presented grey-box model is based on the thermal-electrical analogy. Besides, the model adopts a lumped-parameter approach. The thermal system to be modeled is composed of two main components, each one of them modeled by the thermal capacitors C_1 and C_2. The heating power, p, is generated directly on capacitor C_1. The model also includes a temperature sensor, whose dynamics is not negligible, and it is modeled by an additional capacitor, C_3 (see Fig.1). The state variables selected for the model are the temperatures of the three elements,

$$x = (T_1 \quad T_2 \quad T_3)^{\mathrm{T}}, \tag{1}$$

and, the corresponding state-space representation is

$$\dot{x} = f_\theta(x,u) = \begin{pmatrix} -\frac{T_1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{T_2}{C_1R_2}+\frac{p}{C_1}+\frac{T_{\mathrm{amb}}}{C_1R_1} \\ \frac{T_1}{C_2R_2}-\frac{T_2}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)+\frac{T_3}{C_2R_4}+\frac{T_{\mathrm{amb}}}{C_2R_3} \\ \frac{T_2}{C_3R_4}-\frac{T_3}{C_3R_4} \end{pmatrix} \tag{2}$$

Note that the dynamical model obtained, despite being linear with respect to the state variables and the input, is nonlinear with respect to the parameters.

Identifiability Analysis

Structural identifiability is a theoretical property that depends exclusively on the parameterization of the model. There are several definitions of identifiability in the literature [3]-[5]. Let us consider the previously presented model as a general input-affine dynamical model,

$$\Sigma_\theta : \begin{cases} \dot{x}(t) = \phi_\theta(x(t)) + g_\theta(x(t))u(t) \\ y(t) = h_\theta(x(t)) \\ x(t_0) = x_0(\theta) \end{cases} \tag{3}$$

The dynamical system Σ_θ and the initial state $x_0(\theta)$ define an input-output map of the form:

$$IO_{(\Sigma_\theta, x_0(\theta))} = \{u(t)\} \mapsto \{y(t)\}, t \in [t_0, t_f], \tag{4}$$

such that for each admissible input, the system returns an output. The system is said to be globally structurally identifiable (**g.s.i.**) if there is a one-to-one relationship between the set of possible values of the parameter vector and the set of possible input-output maps. That is, if it is satisfied that

$$IO_{(\Sigma_\theta, x_0(\theta))} = IO_{(\Sigma_{\tilde{\theta}}, x_0(\tilde{\theta}))} \Leftrightarrow \theta = \tilde{\theta}\,. \qquad (5)$$

There are several methods proposed in the literature to assess identifiability [5]-[9]. In this paper we use the method of the local state isomorphism. This method postulates that if it is possible to find a diffeomorphism, φ, between the state spaces of two different representations of the system, then the theorem establishes that both representations correspond to the same input-output map. If, furthermore, the existence of the diffeomorphism is conditioned by an equality relation between the parametric sets, then the system is **g.s.i.**. This method is applied for two cases, in one of them the state can be fully measured, while in the other only one state variable can be measured.

A. Case I. Complete state measurement

Assuming that all state variables can be measured, the resolution of the diffeomorphism gives rise to a series of relations[1] of the type $\alpha_i(\theta) = \alpha_i(\tilde{\theta})$ between the parameters. These relations are the following:

$$\alpha_1(\theta) = B_1 \qquad (6)$$
$$\alpha_2(\theta) = B_1(G_1 + G_2) \qquad (7)$$
$$\alpha_3(\theta) = B_1 G_2 \qquad (8)$$
$$\alpha_4(\theta) = B_2 G_2 \qquad (9)$$
$$\alpha_5(\theta) = B_2 G_4 \qquad (10)$$
$$\alpha_6(\theta) = B_2(G_2 + G_3 + G_4) \qquad (11)$$
$$\alpha_7(\theta) = B_3 G_4 \qquad (12)$$

The set of equations imply that $\theta = \tilde{\theta}$. Therefore, the model is **g.s.i.** if the complete state is measured.

B. Case II. Partial state measurement

In practice it is not always possible to measure the complete state. As an example, we assume that we only have measurements of T_1. The resolution of this diffeomorphism gives rise to a set of relations of the type $\beta_i(\theta) = \beta_i(\tilde{\theta})$, as follows:

$$\beta_1(\theta) = B_1 \qquad (12)$$
$$\beta_2(\theta) = B_1(G_1 + G_2) \qquad (14)$$
$$\beta_3(\theta) = B_1 B_2 G_2^2 \qquad (15)$$
$$\beta_4(\theta) = B_2(G_2 + G_3 + G_4) \qquad (16)$$
$$\beta_5(\theta) = B_2 G_4 \qquad (17)$$
$$\beta_6(\theta) = B_3 G_4 \qquad (18)$$

In this case, the set of equations does not imply $\theta = \tilde{\theta}$. Thus, the model is not **g.s.i**. measuring only T_1.

Further Results and Discussion

A. Case I. Complete state measurement

In the case of measuring the complete state, the identifiability analysis shows that the proposed model is identifiable. Therefore, a parametric identification process is proposed to determine the value of the unknown parameters of the model. In this work we have used as a cost function a weighted sum of the root mean squared errors of the three temperatures. MATLAB mathematical software is used to solve the optimization problem. The main numerical results of the parametric identification process are included in Tables I and II. The adjustment of temperatures is shown in Figs. 2, 3 and 4.

B. Case II. Partial state measurement

In the case of measuring only one state variable the problem of parametric identification will be approached from another perspective. In this regard, it is intended to show the identifiability result obtained for the case B. To do this, we search two different parametric sets that give rise to the same input-output map for the measured variable. The numerical values of these parameters are listed in Table III. The simulation results of the model dynamics for the two sets of different parameters, θ_{A} and θ_{B}, are represented in Figs. 5, 6 and 7.

Conclusions

In this work we have presented an identifiability study of a heat transfer system for two different situations. In the first situation we have shown that the system is identifiable if all temperatures are measured. This ensures the existence of a unique parametric set for each possible dynamics of the system. In the second scenario, we show that the system is not identifiable if only the temperature T_1 is measured. A parametric fit can lead to a model that correctly reflects the measured temperature dynamics, but there is no guarantee that the estimated parameters make physical sense.

[1] Note that for algebraic simplicity, the inverses of resistances and capacitances are used, that is, $G_i = R_i^{-1}$ and $B_i = C_i^{-1}$.

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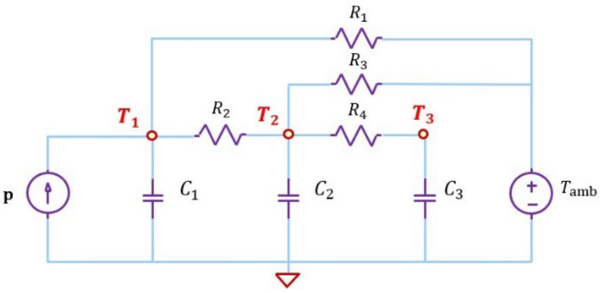

Fig. 1. Schematic representation of the lumped-parameter structure

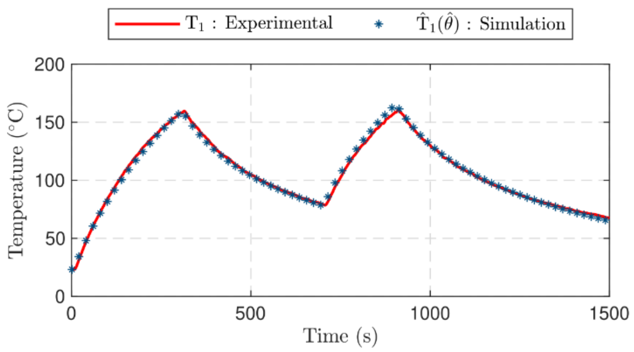

Fig. 2. Temporal evolution of temperature T_1

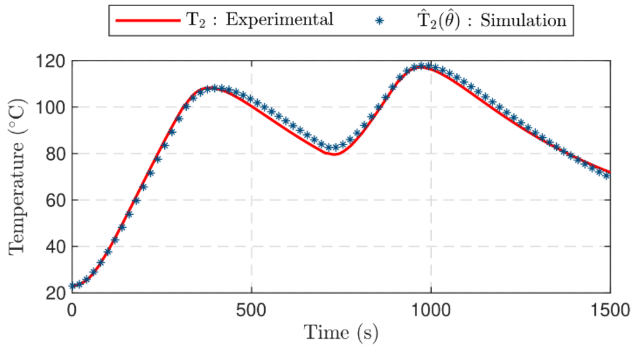

Fig. 3. Temporal evolution of temperature $\boldsymbol{T_2}$

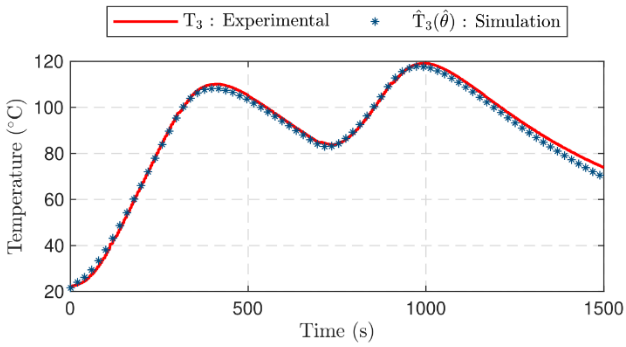

Fig. 4. Temporal evolution of temperature $\boldsymbol{T_3}$

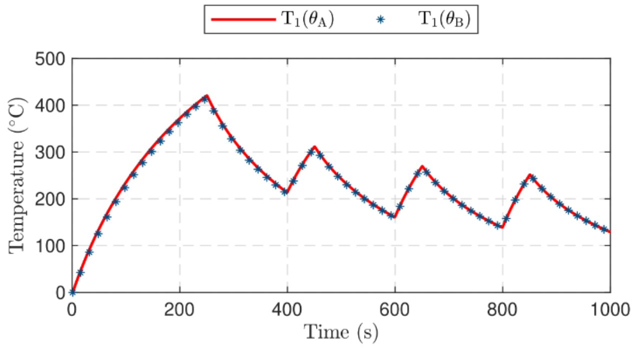

Fig. 5. Temporal evolution of the simulated temperature T_1 for the parametric sets $\boldsymbol{\theta}_A$ and $\boldsymbol{\theta}_B$

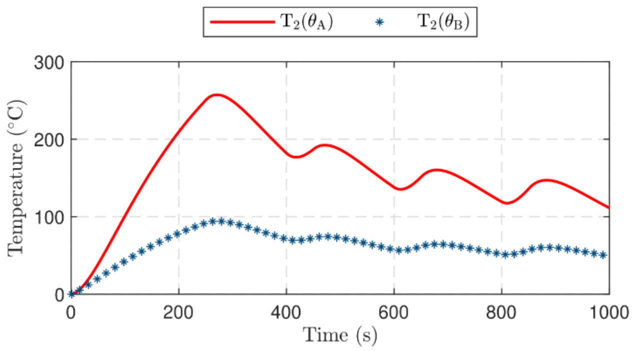

Fig. 6. Temporal evolution of the simulated temperature T_2 for the parametric sets $\boldsymbol{\theta}_A$ and $\boldsymbol{\theta}_B$

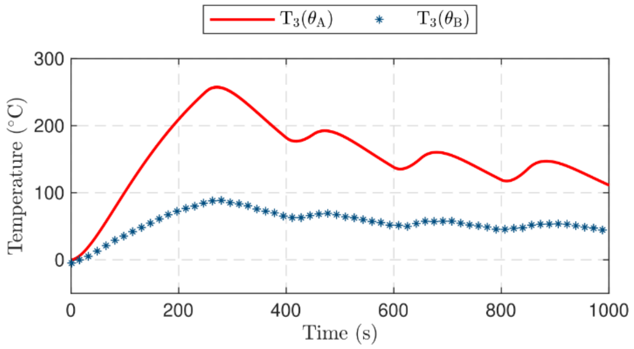

Fig. 7. Temporal evolution of the simulated temperature $\boldsymbol{T_3}$ for the parametric sets $\boldsymbol{\theta}_{\mathrm{A}}$ and $\boldsymbol{\theta}_{\mathrm{B}}$

Table I. Parameter Identification Results. Errors

Temperature	RMSE (ºC)	T_{mean}(ºC)	$\frac{\mathrm{RMSE}}{T_{\mathrm{mean}}}$ (%)
T_1	2.15	105.96	2.03
T_2	2.06	88.5	2.33
T_3	1.89	88.65	2.13

Table II. Parameter Identification Results. Parameter Set

Parameter	Inital value, θ_0	Optimum value, θ^*
R_1	1 K/W	1.2 K/W
R_2	1 K/W	0.835 K/W
R_3	1 K/W	4.35 K/W
R_4	1 K/W	0.1 K/W
C_1	350 J/K	380 J/K
C_2	180 J/K	213.4 J/K
C_3	0.9 J/K	0.995 J/K

Table III. Numerical Values of Parameter Sets $\boldsymbol{\theta}_{\mathrm{A}}$ and $\boldsymbol{\theta}_{\mathrm{B}}$

Parameter	θ_{A}	θ_{B}	Units
R_1	1	-0.3008	K/W
R_2	0.5	0.1581	K/W
R_3	1	0.0442	K/W
R_4	0.1	0.01	K/W
C_1	300	300	J/K
C_2	150	1500	J/K
C_3	1	10	J/K