

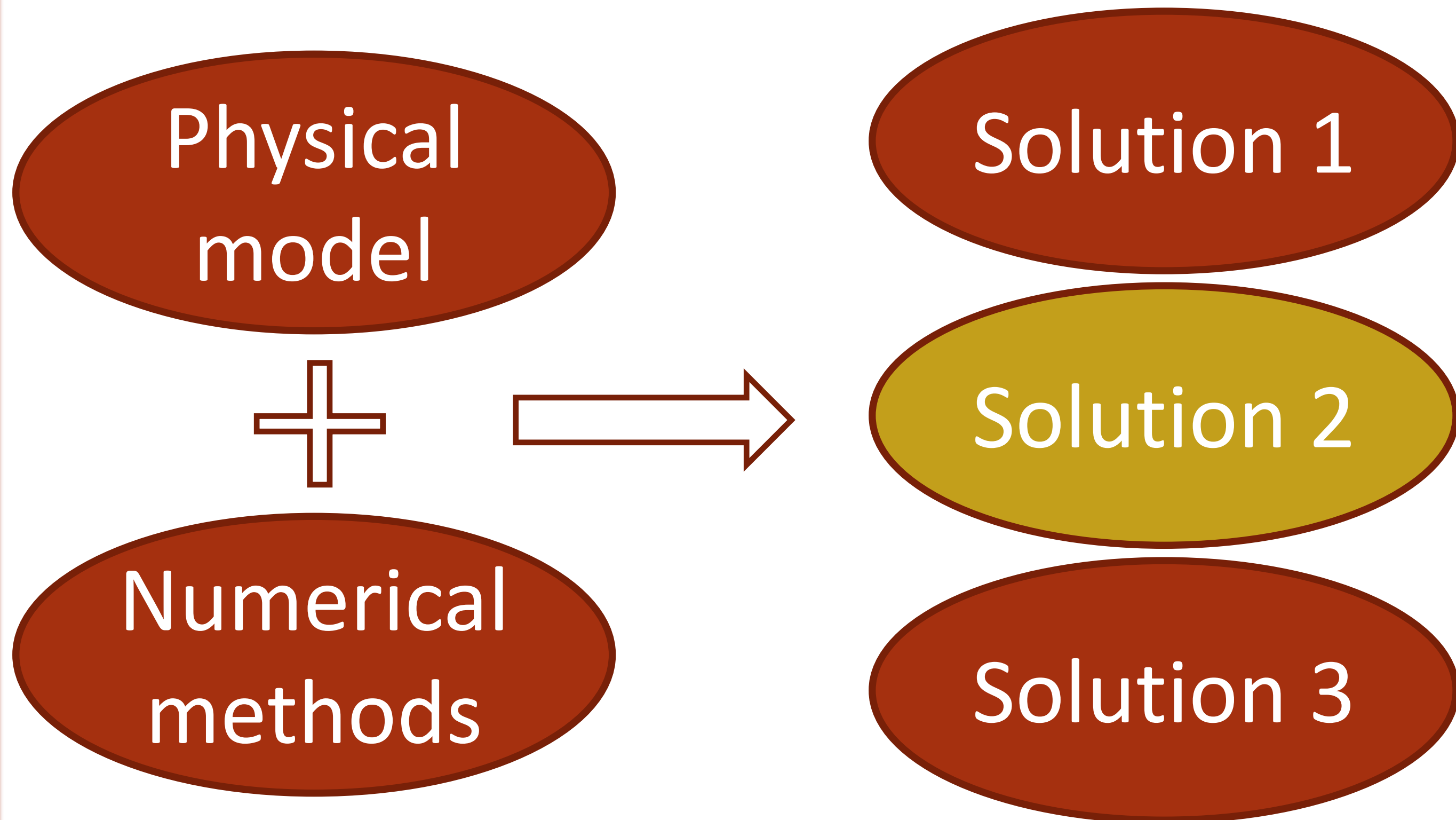
# Improving Sonic Rarefactions in Elastic Vessels: Application to the Tourniquet Manoeuvre

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A method to calculate 1D Flow in veins with varying section needs corrections or it can otherwise produce unphysical results.

## Entropy Corrections

An *entropy condition* identifies the physically correct solutions among all the solutions produced by a numerical method.



## Source term correction

Fluid flow is solved with conservation laws (1), an integral over a control volume

$$\int_{-x_L}^{x_R} \int_0^{\Delta t} \left( \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} - S \right) dt dx = 0 \quad (1)$$

Integrating  $U$  and  $F$  is "easy", but  $S$  needs to be approximated by a constant value  $\bar{s}$  in each timestep. There are different choices

- **Differential:** Using the hydrostatic force on the walls at each cell.

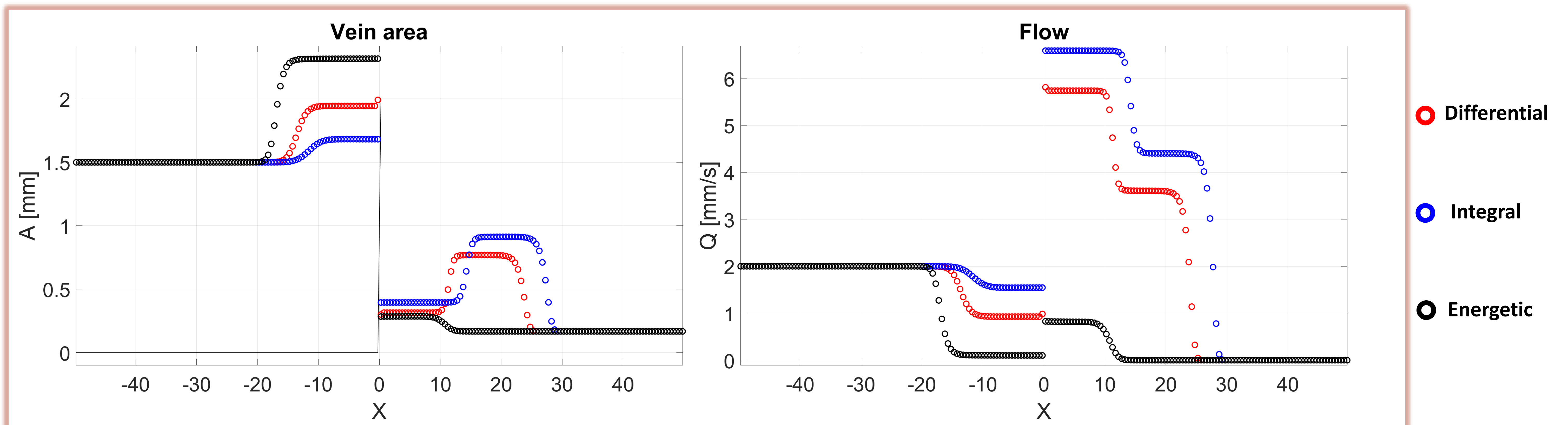
$$\bar{S} = -g\tilde{A}\Delta z \quad (2)$$

- **Integral:** The pressure exerted by the wall discontinuity is integrated on the section.

$$\bar{S} = \int_{z-r}^{z+r} -g\tilde{A}dz' \quad (3)$$

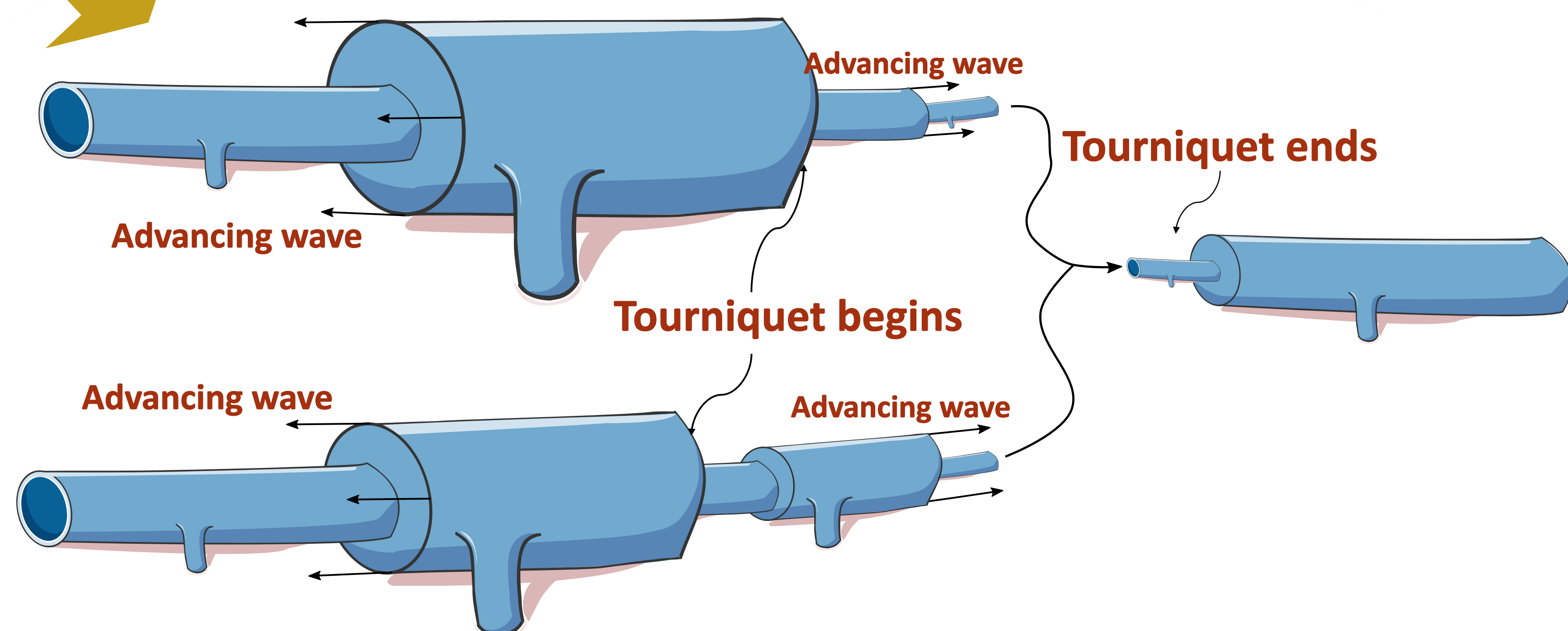
- **Energetic:** The weighted average of the Differential and Integral formulations is taken according to energy conservation arguments.

## Numerical results



Energy conservation formulation

Integral/Differential formulation



## Conclusions

- The differential and integral formulations of the source term are insufficient when dealing with large discontinuities.
- Formulating the source term based on energy conservation arguments often produces better results, except when dissipation

## Future work

- Better source term integration.
- Extension to vessel junctions.
- Calculations of stress on venous walls.
- Addition of dissipation.

## References

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