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Structural Identifiability Applied to a Heat Transfer System

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Introduction

- Reducing energy consumption improving the efficiency of the thermal process.
- A useful tool to improve the efficiency is building a dynamical model.
- The building of a model aimed at energy improvement depends:
  - The degree of detail
  - The information used in the construction process

  } Compromise
- The optimal solution is a hybrid modeling between models based on basic principles whose parameters are characterized from observations (**GREY-BOX** model see [1]).
- This modeling approach requires identification procedures in order to estimate the unknown parameters.

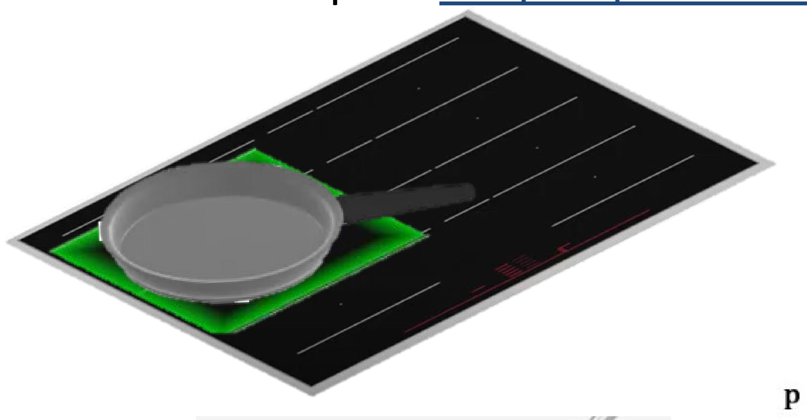
- Both if the parameters have physical significance or it is necessary predict the dynamics of state variables that cannot be measured, it is essential to perform an **IDENTIFIABILITY ANALYSIS**.
- Despite the importance of identifiability, its analysis has been largely overlooked in the vast majority of works on dynamic modeling.
- The goal of this work is highlighting the role played by the identifiability property applied to a thermal system given its relevance in improving energy system.

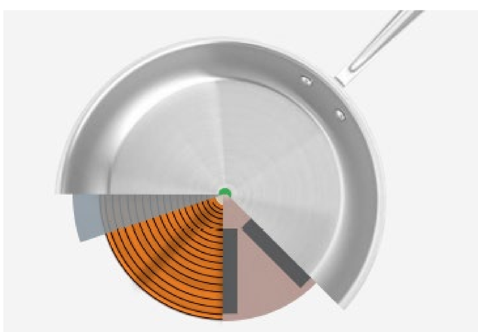
Dynamical Model

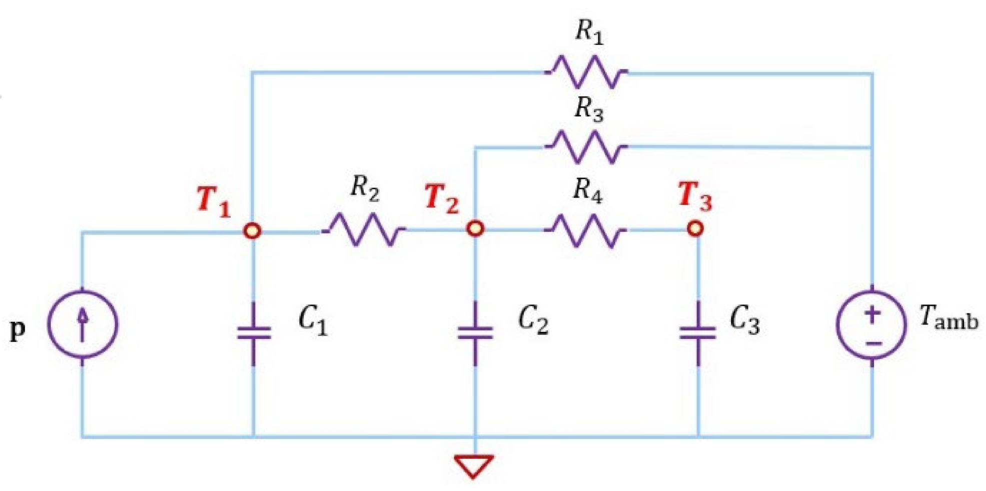
The thermal system to be modeled consists of a **pan**, the **induction hob glass** and the **inductor**.

The model also includes a **temperature sensor**.

- The presented grey-box model is based on the thermal-electrical analogy.
- The model adopts a lumped-parameter approach.







- **State-space representation**

$$\dot{x} = f_\theta(x,u) = \begin{pmatrix} -\frac{T_1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{T_2}{C_1R_2}+\frac{p}{C_1}+\frac{T_{amb}}{C_1R_1} \\ \frac{T_1}{C_2R_2}-\frac{T_2}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)+\frac{T_3}{C_2R_4}+\frac{T_{amb}}{C_2R_3} \\ \frac{T_2}{C_3R_4}-\frac{T_3}{C_3R_4} \end{pmatrix}$$

- The state variables:

$$x = (T_1 \quad T_2 \quad T_3)^{\mathrm{T}}$$

- Input of the system:

$$u = (p \quad T_{amb})^{\mathrm{T}}$$

- Parameters:

$$\boldsymbol{\theta} = (R_1 \quad R_2 \quad R_3 \quad R_4 \quad C_1 \quad C_2 \quad C_3)^{\mathrm{T}}$$

Note that the dynamical model obtained, despite being linear with respect to the state variables and the input, is nonlinear with respect to the parameters.

Identifiability Analysis

Structural identifiability is a theoretical property that depends exclusively on the parameterization of the model. There are several definitions [2]-[3].

Considering the presented model as a general input-affine dynamical model,

$$\Sigma_\theta : \begin{cases} \dot{x}(t) = \phi_\theta(x(t)) + g_\theta(x(t))u(t) \\ y(t) = h_\theta(x(t)) \\ x(t_0) = x_0(\theta) \end{cases}$$

The dynamical system Σ_θ and the initial state $x_0(\theta)$ define an input-output map:

$$IO_{(\Sigma_\theta, x_0(\theta))} = \{u(t)\} \mapsto \{y(t)\}, t \in [t_0, t_f],$$

such that for each admissible input, the system returns an output.

The system is globally structurally identifiable (**g.s.i.**) if there is a one-to-one relationship between the set of possible values of the parameter vector and the set of possible input-output maps.

$$IO_{(\Sigma_\theta, x_0(\theta))} = IO_{(\Sigma_{\tilde{\theta}}, x_0(\tilde{\theta}))} \Leftrightarrow \theta = \tilde{\theta}\,.$$

In this work we use to assess identifiability the method of the **local state isomorphism** (**LSIT**).

LSIT postulates that exist a diffeomorphism, $\boldsymbol{\varphi}$, between the state spaces of two different representations of the system, then both representations correspond to the same input-output map. If, furthermore, the existence of $\boldsymbol{\varphi}$ is conditioned by an equality relation between the parametric sets, then the system is **g.s.i.**

A. Case I. Complete State Measurement

Applying LSIT assuming that all states are measured, we obtain a series of relations $\boldsymbol{\alpha_i(\theta) = \alpha_i(\tilde{\theta})}$,

$$\boldsymbol{\alpha_i(\theta)}: \begin{cases} \alpha_1(\theta) = B_1 \\ \alpha_2(\theta) = B_1(G_1+G_2) \\ \alpha_3(\theta) = B_1G_2 \\ \alpha_4(\theta) = B_2G_2 \\ \alpha_5(\theta) = B_2G_4 \\ \alpha_6(\theta) = B_2(G_2+G_3+G_4) \\ \alpha_7(\theta) = B_3G_4 \end{cases}$$

The equations imply that $\boldsymbol{\theta = \tilde{\theta}}$. Therefore, the model is **g.s.i.** if the complete state is measured.

B. Case II. Partial State Measurement

Applying LSIT assuming only T_1 is measured, we obtain a series of relations $\boldsymbol{\beta_i(\theta) = \beta_i(\tilde{\theta})}$,

$$\boldsymbol{\beta_i(\theta)}: \begin{cases} \beta_1(\theta) = B_1 \\ \beta_2(\theta) = B_1(G_1+G_2) \\ \beta_3(\theta) = B_1B_2G_2^2 \\ \beta_4(\theta) = B_2(G_2+G_3+G_4) \\ \beta_5(\theta) = B_2G_4 \\ \beta_6(\theta) = B_3G_4 \end{cases}$$

The equations does not imply that $\boldsymbol{\theta = \tilde{\theta}}$. Thus, the model is **not g.s.i.** measuring only T_1.

Further Results and Discussion

A. Case I. Complete State Measurement

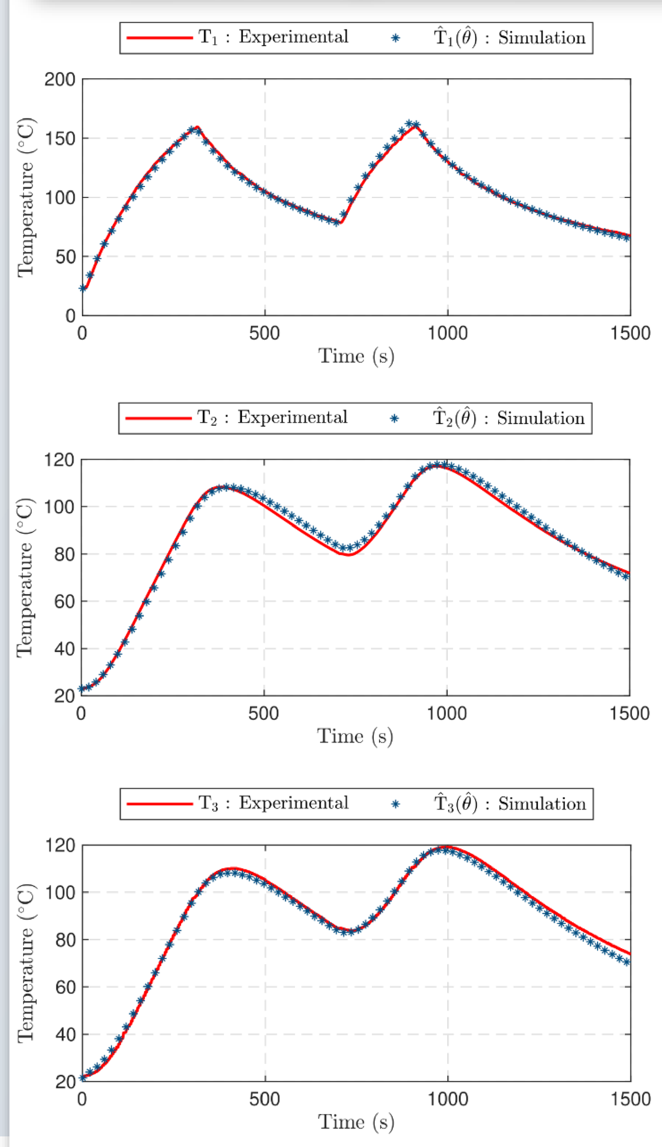


Table I. Parameter Identification Results. Errors

Temperature	RMSE (ºC)	T_{mean}(ºC)	$\frac{RMSE}{T_{mean}}$ (%)
T_1	2.15	105.96	2.03
T_2	2.06	88.5	2.33
T_3	1.89	88.65	2.13

Table II. Parameter Identification Results. Parameter Set

Parameter	Inital value, θ_0	Optimum value, θ^*
R_1	1 K/W	1.2 K/W
R_2	1 K/W	0.835 K/W
R_3	1 K/W	4.35 K/W
R_4	1 K/W	0.1 K/W
C_1	350 J/K	380 J/K
C_2	180 J/K	213.4 J/K
C_3	0.9 J/K	0.995 J/K

B. Case II. Partial State Measurement

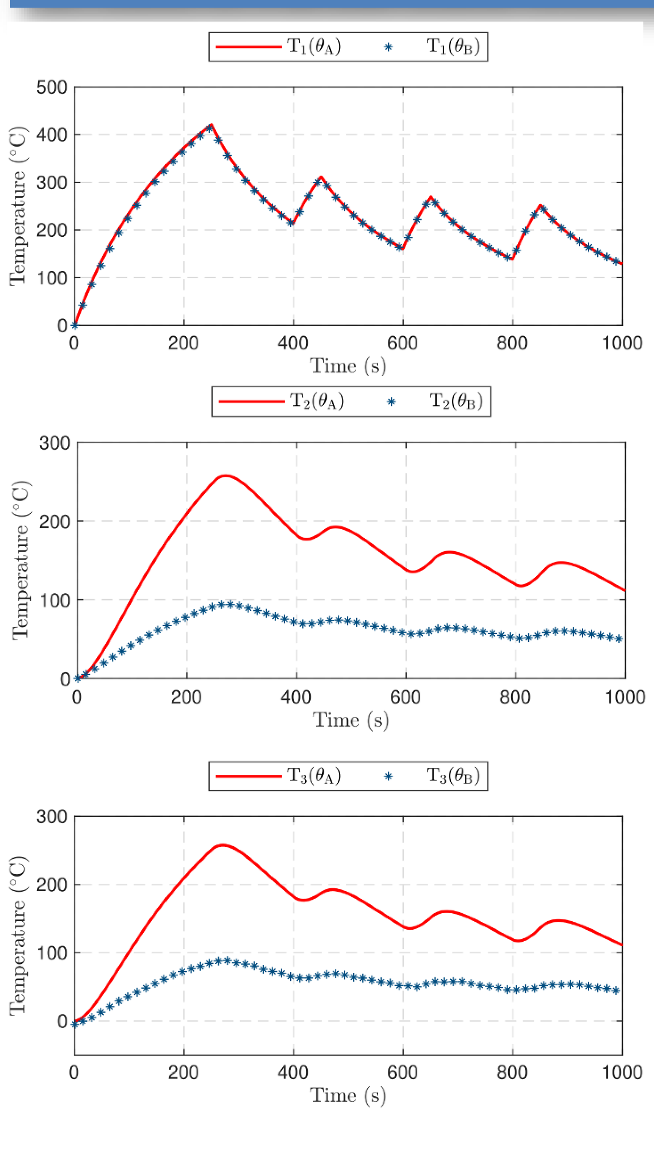


Table III. Numerical Values of Parameter Sets $\boldsymbol{\theta}_A$ and $\boldsymbol{\theta}_B$

Parameter	θ_A	θ_B	Units
R_1	1	-0.3008	K/W
R_2	0.5	0.1581	K/W
R_3	1	0.0442	K/W
R_4	0.1	0.01	K/W
C_1	300	300	J/K
C_2	150	1500	J/K
C_3	1	10	J/K

Conclusions

- In this work we have analyzed the identifiability of a heat transfer system for two cases:
- In the case I the model is **g.s.i.** so this ensures de existence of a unique parametric set for each possible dynamics of the system.
- In the case II we show that the system is **not g.s.i.**. A parametric fit can lead to a model that correctly reflects the measured temperature dynamics, but there is no guarantee that the estimated parameters make physical sense.

References

[1] LUCHI, M. and LORENZINI, M. Control-oriented low-order models for the transient analysis of a domestic electric oven in natural convective mode. Applied Thermal Engineering. 2019, Elsevier.

[2] LJUNG, L. and GLAD, T. On global identifiability for arbitrary model parametrizations. Automatica. 1994, Elsevier.

[3] VADJA, S. and RABITZ, H. State Isomorphism approach to global identifiability of nonlinear systems. IEEE Transactions on Automatic Control. 1989, IEEE.

Acknowledgments

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